

**Faculty of Economics and Business  
Universitas Indonesia**

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**Final Exam  
Odd 2019/2020**

Course	: Mathematics for Economics and Business
Lecturer	: Team
Day/Date	: Tuesday/December 17, 2019
Time	: 3 hours
Instruction	: Closed book, using calculator is permitted

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**Question 1 (25 points)**

Sales value of pearl ( $V$ ) from freshwater shells is an increasing function of time ( $t$ , in year):

$$V(t) = e^{10+t^{0.45}}$$

Assuming that continuous interest rate is a constant 5 per cent per year and no cost incurs in cultivating the freshwater shells:

- a. Find the function of the pearl sales in present value. **(3 points)**
- b. When should the pearls be harvested to get the highest present value of its sales? **(10 points)**
- c. Prove that your calculation result in point (b) is indeed the optimal time to harvest. **(8 points)**
- d. Calculate the maximum sales value of pearl in present value. **(4 points)**

**Question 2 (25 points)**

MatBi is a firm that produces three kinds of product,  $Q_1$ ,  $Q_2$ , and  $Q_3$ . MatBi's cost function is given by:

$$C = 2Q_1^2 + 2Q_2^2 + 2Q_3^2 - Q_1Q_2 - Q_1Q_3 - Q_2Q_3$$

where  $C$  = total cost;  $Q_i$  = quantity of product  $i$  ( $i = 1, 2, 3$  in millions of units)

MatBi tries to maximize its profit, but because of the tight competition, they can only follow the existing price in the market for its products. The current prices of product 1, 2, and 3 (in US Dollar) are 2, 18, and 5 respectively.

- a. How many product 1, 2, and 3 that MatBi needs to produce to maximize its profit? **(10 points)**
- b. Show that the quantity of product 1, 2, and 3 at point (a) will give MatBi a maximum profit. **(10 points)**
- c. Calculate the maximum profit of MatBi. Make sure the units are correct. **(5 points)**

### **Question 3 (25 points)**

Anas has the following utility function ( $U$ ) from consuming good  $X$  and  $Y$ :

$$U(X, Y) = 17X + 20Y - 2X^2 - Y^2$$

Anas' income is Rp22,000 per month, while prices of good  $X$  and  $Y$  are Rp3,000 and Rp4,000 per unit, respectively.

- a. Find the objective and the constraint function of Anas, and then state the Lagrange function for Anas optimization problem. **(3 points)**
- b. How many good  $X$  and  $Y$  that Anas needs to buy and consume in order to get the maximum utility? **(8 points)**
- c. Find the Lagrange multiplier number from the calculation above and interpret the result. **(4 points)**
- d. Calculate the resulting maximum utility of Anas. **(3 points)**
- e. Show that the utility of Anas at point (d) is indeed a maximum utility. **(7 points)**

### **Question 4 (25 points)**

In a competitive market, both consumers and producer gain maximum surplus from buying and selling at equilibrium price. Demand for a good in certain competitive market is:

$$P = D(Q) = 113 - Q^2$$

On the other hand, the supply is given by:

$$P = S(Q) = (Q + 1)^2$$

$P$  and  $Q$  are price and quantity of the good, whereas  $D$  and  $S$  represent its demand and supply.

- a. State the market equilibrium condition, and then find the equilibrium price and quantity. **(5 points)**
- b. Draw a complete diagram showing the market equilibrium condition and surpluses for both consumers and producers. **(5 points)**
- c. Referring to calculation results and diagram at point (a) dan (b), find and calculate the total surplus in that market. **(15 points)**

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